# Compound Interest – The 1/4-Million Dollar Car

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What's the magic formula for investing success? About as close as you'll come to it is the simple "Time Value of Money" (TVM) or the "compound interest" equation, $FV_n=PV(1+r)^n$. This equation and its related formulae serve as key components in decision making on the parts of both individual investors and the managers of our largest corporations.

Put simply, the compound interest formula relates the future value (FV) of the present value (PV) of a dollar when it is invested for a period of years (n) at a given rate of interest (r). For example, after 10 years the future value ($FV_{10}$) of $100 invested at 10% (expressed as 0.10) would be $\$100(1+0.10)^{10}$ or $100 times 1.10 that has been raised to the power of 10. Raising 1.10 to the tenth power means multiplying 1.10 by itself 10 times, such "raising" being expressed by the exponent $^{10}$. The value can be found with almost any inexpensive handheld scientific calculator by employing the "$y^x$" key. Key in 1.10, press the $y^x$ key, then enter 10 and hit the "equals" button (answer: 2.5937). In this case the future value is $100 times 2.5937 or $259.37.

If you don't have a cheap scientific calculator or a lot of time on your hands to do it manually, some older financial books have tables of the exponential values for various interest rates and various numbers of years. Print-outs of these same tables or even programmes that calculate the "future value factor" for you can be found on the Internet by searching for "future value table". (The University of Arizona's website at www.studyfinance.com has a lot of these resources under its link to Lectures on the Time Value of Money, as well as helpful explanations.) It's actually a lot of fun to see what your money can grow to at a certain interest rate over a certain number of years. If you find it really fun, an intermediate level financial calculator such as the Texas Instruments TI-BA II Plus for $45 or so would be a good investment.

But what's so magic about compound interest? Its power lies in its exponential powers. "Simple interest" on the $100 invested for 10 years at 10% would give you $10 a year for 10 years or $100. You would have only $200 in total at the end of that time, not the $259.37 that compound interest generated in the above example. And at the end of 50 years, you would have made $500 in simple interest at that rate of return and have $600 in total. But with compound interest the total would be – wait for it – $11,739. The expo-

**TABLE 1**

**Future Value Factors**

Below is an excerpt from a full table of future value factors, listing only a few of the possible combinations of number of years (n) and interest rates (r) from the formula $FV_n=PV(1+r)^n$. To determine the value of the future value factor $(1+r)^n$, read down the first column "Years" and then across to the appropriate interest rate. Then multiply this figure by the amount of money you are specifying (PV) to get the future value (FV) of that sum. For example, in the accompanying article, 42 years at 7% compounding annually gives a future value factor of 17.1443 (see table below). Multiply that figure by $15,000 and the result is $257,165. The website of one of the major mutual fund companies features a handy FV calculator, among many other calculators, that will do the math for you:
www.mackenziefinancial.com/en/pub/tools/calculators/index.shtml

| Years | 1% | 3% | 5% | 7% | 10% |
|---|---|---|---|---|---|
| 1 | 1.0100 | 1.0300 | 1.0500 | 1.0700 | 1.1000 |
| 5 | 1.0510 | 1.1593 | 1.2763 | 1.4026 | 1.6105 |
| 10 | 1.1046 | 1.3439 | 1.6289 | 1.9672 | 2.5937 |
| 15 | 1.1610 | 1.5580 | 2.0789 | 2.7590 | 4.1772 |
| 20 | 1.2202 | 1.8061 | 2.6533 | 3.8697 | 6.7275 |
| 25 | 1.2824 | 2.0938 | 3.3864 | 5.4274 | 10.8347 |
| 30 | 1.3478 | 2.4273 | 4.3219 | 7.6123 | 17.4494 |
| 35 | 1.4166 | 2.8139 | 5.5160 | 10.6766 | 28.1024 |
| 40 | 1.4889 | 3.2620 | 7.0400 | 14.9745 | 45.2593 |
| 42 | 1.5188 | 3.4607 | 7.7616 | 17.1443 | 54.7637 |
| 45 | 1.5648 | 3.7816 | 8.9850 | 21.0025 | 72.8905 |
| 50 | 1.6446 | 4.3839 | 11.4674 | 29.4570 | 117.3909 |

nential growth of the initial amount (in this case $100 multiplied by 1.10 raised to the 50th power, or 117.39) produces some fantastic sums.

Of course, it's not really magic, only math. What happens with compound interest is that the interest that is earned each year is reinvested for the years following, so that the amount invested keeps growing and growing. The more years that pass, the greater the amount of money invested and earning interest. Simple interest is paid at the same rate every year as compound interest, but the interest earned is not added to the amount invested, which in this case remains $100 forever. A striking illustration of the compounding principle that is sometimes discussed in schools is to place a grain of rice on the corner square of a chess board, and then double that amount (a 100% rate of growth) on the square next to it, then double that on the next square and so forth. You can guess that at this 100% "interest" rate (expressed as 1.00) for 64 squares (raised to the 64th power) the equation $FV_{64} = PV(1+1.00)^{64}$ would provide more than enough rice to feed the entire planet!

How can this math be put to use to invest profitably? First, one must avoid losing capital permanently. A permanent loss of capital means that there will be no compounding interest at all on the capital amount that has been lost. And compound interest is what is important. Even if the capital amount lost was small, it could have generated a huge amount of compound interest over the years. While the inevitable ups and downs in market value of solid, blue-chip investments are tolerable, risking a permanent loss of capital on a poorly researched investment can be disastrous. Second, a higher rate of return on investments increases the effect of compounding in the long term – as long as chasing that return does not risk losing capital!

Third, and most important, the magic only works if a lot of time passes. Starting early and having the patience to hold solid investments for several decades is the key to success. A relatively small amount of money invested can produce a huge amount of money if left to accumulate over a long period of time. Starting to save a little money early on is the same as investing a lot of money much later. Or to put it another way, patience is a low-risk, high-return investment strategy.

To illustrate, let's consider the case of a young man of 23 who is given a gift of $50,000. He spends $20,000 of it and he invests $30,000 for retirement in securities having an average annual return of 7% (3% dividends plus 4% annual capital gains). But the following year he sees a hot used car for $15,000 and snaps it up, taking the money from his new retire-

**TABLE 2**

**The Magic Formula Revealed**

A formula such as that for TVM, or the Time Value of Money, $FV_n=PV(1+r)^n$, can sometimes make more sense if its derivation is explained. As noted in the article, PV is the Present Value of the initial deposit, n is the Number of years (or periods, if not a year) that the PV is invested, r is the interest Rate used, and FV is the Future Value of the money after n number of years have passed.

Common sense would tell you that at the end of year one, you would get back your original deposit plus the original deposit multiplied by the interest rate (simple interest). This is expressed mathematically as $FV_1 = PV + PV(1+r)^1$. A more compact way to state this same thing is $FV_1 = PV(1+r)^1$ and we will use this form from now on. (The exponent 1, which does not raise the figure it is used with, does not normally appear, but it is employed here for the sake of consistency.)

For the end of the second year calculation of $FV_2$, common sense also would say that the new "initial" PV at the start of that second year is what the $FV_1$ was at the end of the first year, or $PV(1+r)^1$. So, if we substitute $PV(1+r)^1$ for PV in the second year formula and multiply it by the same interest rate 1+r for that year as we did for the first year, we get the equation $FV_2 = PV(1+r)^1 (1+r)^1$. Another way to write the $(1+r)^1(1+r)^1$ part is as $(1+r)^2$, or (1+r) squared, since it is an amount multiplied by itself. The second year equation is thus $FV_2 = PV(1+r)^2$. For the third year, one would multiply the PV and the two (1+r) amounts by a third (1+r) to get $FV_3 = PV(1+r)^3$. And so on, for every subsequent year, so that each $FV_n$ would have a corresponding $PV(1+r)^n$.

A straightforward algebraic transformation of the TVM equation is used for "Discount Rate" calculations to determine the present value of a sum of money to be received (or expended) in the future. For instance, if you expect $100 (FV) in 10 years, how much would it be worth in buying power in "today's dollars" (PV) if inflation were running at 5% annually over that period? The transformed TVM equation would be $PV = FV_{10} / (1+r)^{10}$, or $100/(1.05)^{10}$, or 100/1.6289, the end result being $61.39. Present Value tables from the Internet will provide the needed factors, or again one can employ a calculator to do the exponential arithmetic.

You may see the equations for calculating annuity payments and for accumulating monthly deposits over time in the same context as TVM calculations. That's because they contain the basic TVM, or compound interest formula. Tables of factors are also available for annuity and periodic savings calculations, or the "annuity row" of keys on a basic financial calculator can do the math quickly.

ment account. He did not want to wait a couple of years to save up and buy a car like it later.

How much did that car ultimately cost him? Its "present value" was $15,000. But its "future value" in 42 years at his retirement age of 65, according to our formula $FV_{42} = PV(1+r)^{42}$ is much higher. How much higher? It would be $15,000 times 1.07 raised to the power of 42, or $15,000 times 17.1443 (factor given by a future value table or the $y^x$ key of a calculator) for a total cost of $257,165. No doubt he didn't realize at the time what an expensive auto he was buying!

For that man, however, all is not lost. He will still have $257,165 waiting for him at retirement even if he never contributes again to his retirement account. Think how much worse off those people will be who do not start saving early. They would have to save a pile and a half to equal this amount. And even worse off will be those who are in debt and have the magic, black magic in this case, of compound interest working against them, slowly drowning them in easy credit rates of 10% or more.

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